Adding and Subtracting with Rational Expressions

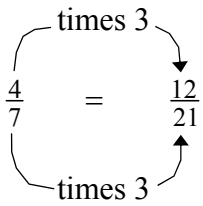
In preparation for adding and subtracting with rational expressions, let us review the problem of finding the missing numerator. See below.

$$\frac{4}{7} = \frac{?}{21}$$

Your eyes see 7 as one denominator and 21 as the other denominator.

Using what your eyes see (the 7 and 21), you can determine that the denominator, 7, has been multiplied by 3, and, to keep the two fractions equivalent, the numerator, 4, above the denominator 7, must also be multiplied by 3.

So the new numerator is 12. See below.



Now let us look at the same problem a different way. See below.

$$\frac{4}{7} = \frac{?}{3 \cdot 7}$$

Notice that the denominator, 21, has been replaced by its prime factorization, $3 \cdot 7$.

Now, the advantage of having the prime factorization, $3 \cdot 7$, in place of 21 is that your eyes see the 3 next to the 7.

So, you do not have to ask, "What do I multiply 7 by to get 21?"

You see with your eyes what you multiplied the 7 by to get $3 \cdot 7$.

Do you see with your eyes what you multiply 5 by to get $8 \cdot 5$?

Do you see with your eyes what you multiply y by to get xy?

Completing the problem above, we get:

$$\frac{4}{7} = \frac{3 \cdot 4}{3 \cdot 7}$$

$$\frac{4}{7} = \frac{12}{21}$$

Once again, the missing numerator is 12.

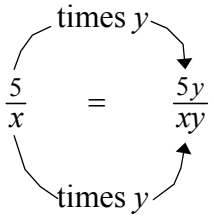
Now find the missing numerator below.

You fill in this missing numerator yourself.

$$\frac{5}{x} = \frac{\quad}{xy}$$

Have you done it?

Look at the following:



The missing numerator is $5y$.

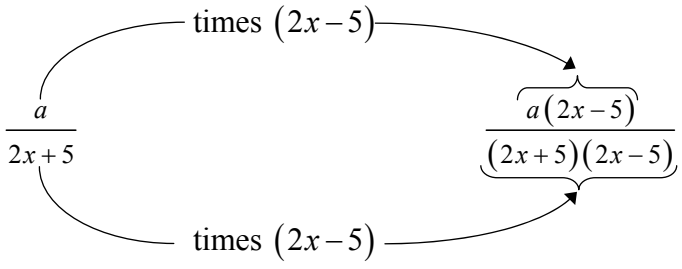
Now find the missing numerator below.

You fill in this missing numerator by yourself.

$$\frac{a}{2x+5} = \frac{\quad}{(2x+5)(2x-5)}$$

Have you done it?

Look at the following:



The missing numerator is $a(2x-5)$.

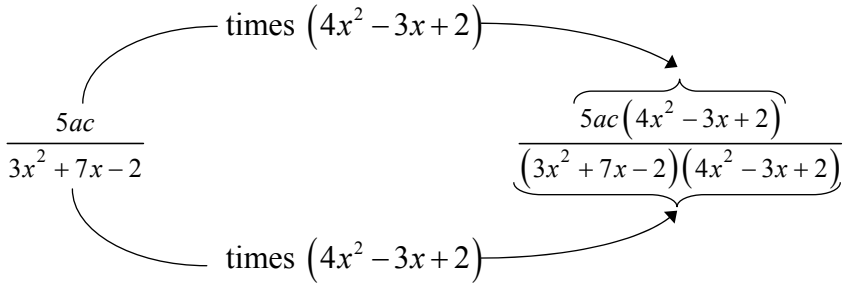
Now find the missing numerator below.

$$\frac{5ac}{3x^2+7x-2} = \frac{\quad}{(4x^2-3x+2)(3x^2+7x-2)}$$

You fill in the missing numerator by yourself.

Have you done it?

Look at the following:



The missing numerator is $5ac\left(4x^2-3x+2\right)$.