Life of Fred

Beginning Algebra

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What Algebra Is All About

When I first started studying algebra, there was no one in my family who could explain to me what it was all about. My Dad had gone through the eighth grade in South Dakota, and my Mom never mentioned to me that she had ever studied any algebra in her years before she took a job at Planter's Peanuts in San Francisco.

My school counselor enrolled me in beginning algebra, and I showed up to class on the first day not knowing what to expect. On that day, I couldn't have told you a thing about algebra except that it was some kind of math.

In the first month or so, I found *I liked algebra better than* . . .

✓ physical education, because there were never any fist-fights in the algebra class.

✓ English, because the teacher couldn't mark me down because he or she didn't like the way I expressed myself or didn't like my handwriting or didn't like my face. In algebra, all I had to do was get the right answer and the teacher had to give me an A.

✓ German, because there were a million vocabulary words to learn. I was okay with der Finger which means *finger*, but besetzen, which means to occupy (a seat or a post) and besichtigen, which means to look around, and besiegen, which means to defeat, and the zillion other words we had to memorize by heart were just too much. In algebra, I had to learn how to *do stuff* rather than just memorize a bunch of words. (I got C's in German.)

✓ biology, because it was too much like German: memorize a bunch of words like mitosis and meiosis. I did enjoy the movies though. It was fun to see the little cells splitting apart—whether it was mitosis or meiosis, I can't remember.

So what's algebra about? Albert Einstein said, "Algebra is a merry science. We go hunting for a little animal whose name we don't know, so we call it x. When we bag our game, we pounce on it and give it its right name."

What I think Einstein was talking about was solving something like $3x - 7 = 11$ and getting an answer of $x = 6$.

But algebra is much more than just solving equations. One way to think of it is to consider all the stuff you learned in six or eight years of studying arithmetic: adding, multiplying, fractions, decimals, etc.   Take all of that and stir in one new concept—the idea of an "unknown," which we like to call "x." It's all of arithmetic *taken one step higher*.

Adding that little "x" makes a big difference. In arithmetic, you could answer questions like: If you go 45 miles per hour for six hours, how far have you gone? In algebra, you may have started your trip at 9 a.m. and have traveled at 45 miles per hour and then, after you've traveled half way to your destination, you suddenly speed up to 60 miles per hour and arrive at 5 p.m. Algebra can answer: At what time did you change speed? That question would "blow away" most arithmetic students, but it is a routine algebra problem (which we solve in chapter four).

Many, many jobs require the use of algebra. Its use is so wide-spread that virtually every university requires that you have learned algebra before you get there. Even English majors, like my daughter Margaret, had to learn algebra before going to a university.

I also liked algebra because there were no term papers to have to write. After I finished my algebra problems I was free to go outside and play. Margaret had to stay inside and type all night. A lot of English majors seem to have short fingers (der Finger?) because they type so much.

A Note to Students

Hi! This is going to be fun.

When I studied algebra, my teacher told the class that we could reasonably expect to spend 30 minutes per page to master the material in the old algebra book we used. With the book you are holding in your hands, you will need two reading speeds: 30 minutes per page when you're learning algebra and whatever speed feels good when you're enjoying the life adventures of Fred.

Our story begins on the day before Fred's sixth birthday. Start with chapter one, and things will explain themselves nicely.

After 12 chapters, you will have mastered all of beginning algebra.

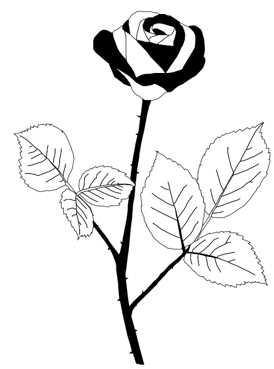
Just before the Index is the **A.R.T.** section, which very briefly summarizes much of beginning algebra. If you have to review for a final exam or you want to quickly look up some topic eleven years after you've read this book, the **A.R.T.** section is the place to go.

Contents

Chapter One

Numbers & Sets

He stood in the middle of the largest rose garden he'd ever seen. The sun was warm and the smell of the roses made his head spin a little. Roses of every kind surrounded him. On his left was a patch of red roses: *Chrysler Imperial* (a dark crimson); *Grand Masterpiece* (bright red); *Mikado* (cherry red). On his right were yellow roses: *Gold Medal* (golden yellow); *Lemon Spice* (soft yellow). Yellow roses were his favorite.



Up ahead on the path in front of him were white roses, lavender roses, orange roses and there was even a blue rose.

Fred ran down the path. In the sheer joy of being alive, he ran as any healthy five-year-old might do. He ran and ran and ran.

At the edge of a large green lawn, he lay down in the shade of some tall roses. He rolled his coat up in a ball to make a pillow.

Listening to the robins singing, he figured it was time for a little snooze. He tried to shut his eyes.

They wouldn't shut.

Hey! Anybody can shut their eyes. But Fred couldn't. What was going on? He saw the roses, the birds, the lawn, but couldn't close his eyes and make them disappear. And if he couldn't shut his eyes, he couldn't fall asleep.

You see, Fred was dreaming. He had read somewhere that the only thing you can't do in a dream is shut your eyes and fall asleep. So Fred *knew* that he was dreaming and that gave him a lot of power.

He got to his feet and waved his hand at the sky. It turned purple with orange polka dots. He giggled. He flapped his arms and began to fly. He settled on the lawn again and made a pepperoni pizza appear.

In short, he did all the things that five-year-olds might do when they find themselves King or Queen of the Universe.

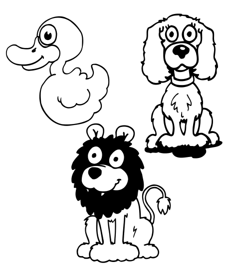
And soon he was bored. He had done all the silly stuff and was looking around for something constructive to do. So he lined up all the roses in one long row.

They stretched out in a line in both directions going on forever. Since this was a dream, he could have an unlimited (**infinite**) number of roses to play with.

When Fred was three years, old he had spent some time studying physics and astronomy. He had learned that nothing in the physical universe was infinite. Everything was **finite** (limited). Every object could travel only at a finite speed. Even the number of atoms was finite. One book estimated that there are only 10^{79} atoms in the observable universe. 10^{79} means 10 times 10 times 10 . . . a total of 79 times, which is 10,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000, 000,000,000,000,000,000,000,000,000,000,000, which is a lot of atoms. (The "79" is an exponent—something we'll deal with in detail later.)

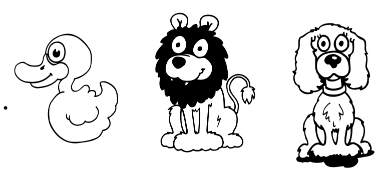
Now that he had all the roses magically lined up in a row, he decided to count them. Math was one of Fred's favorite activities.

Now, normally when you've got a bunch of stuff in a pile to count,



(← *These are Fred's dolls that he used to play with when he was a baby.*)

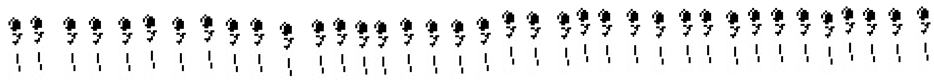
you line them up ....



and start on the left and count them 1 2 3

But Fred couldn't do that with the roses he wanted to count. There were too many of them. He couldn't start on the left as he did with his dolls. Dolls are easy. Roses are hard.

So how do you count them? There wasn't even an obvious "middle" rose to start at. In some sense, every rose is in the middle since there is an infinite number of roses on each side of every rose. So Fred



1

just selected a rose and called it "1." From there it was easy to start counting . . . . 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15. . . .

This **set** (collection, group, bunch) of numbers {1, 2, 3, 4, 5, . . .} is called the **natural numbers**. At least, Fred figured, with the natural numbers he could count half of all the roses.

What to do? How would he count all the roses to the left of "1"? Then Fred remembered the movies he'd seen where rockets were ready for blastoff. The guy in the tower would count the seconds to blastoff: "Five, four, three, two, one, zero!" So he could label the rose just to the left of the rose marked "1" as "0."

This new set, {0, 1, 2, 3, . . . } is called the **whole numbers**. It's easy to remember the name since it's just the natural numbers with a "hole" added. The numeral zero does look like a hole.

A set doesn't just have to have numbers in it. Fred could gather the things from his dream and make a set: {roses, lawn, birds, pizza}. The funny looking parentheses are called braces. Braces are used to enclose sets.

Left brace: { Right brace: }

On a computer keyboard, there are actually three types of grouping symbols: Parentheses: ( )
Braces: { } and
Brackets: [ ]

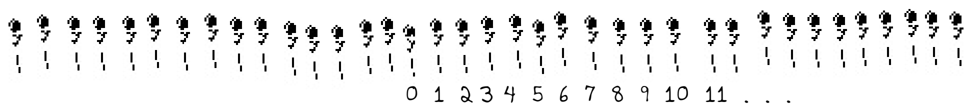
In algebra, braces are used to list the members of a set, while both parentheses and brackets are used around numbers. For example, you might write (3 + 4) + 9 or [35 – 6] + 3.

Braces and brackets both begin with the letter "b" and to remember which one is braces, think of braces on teeth. Those braces are all curly and twisty.

In English classes, parentheses and brackets are not treated alike. If you want to make a remark in the middle of a sentence (as this sentence illustrates), then you use parentheses (as I just did).

Brackets are used when you're quoting someone and you want to add your own remarks in the middle of their quote: "Four score and seven [87] years ago. . . ."

But brackets and parentheses weren't going to help Fred with counting all those roses. The whole numbers only got him this far:



What he needed were some new numbers. These new numbers would be numbers that would go to the left of zero. So years ago, someone invented negative numbers: minus one, minus two, minus three, minus four. . . .

<u>Some notes on negative numbers:</u>

♪#1: It would be a drag to have to write this new set as {. . . minus 3, minus 2, minus 1, 0, 1, 2, . . .}, or even worse, to write {. . . negative 3, negative 2, negative 1, 0, 1, 2, . . .}, so we'll invent an abbreviation for "minus." What might we use?

How about screws? The two most common kinds look like ⊕ (Phillips screws) and ⊖ (slotted screws). Okay. Our new number system will be written {. . . –3, –2, –1, 0, +1, +2, +3, +4, . . . }. We'll call this new set **the integers**.

A.R.T.

All **R**eorganized **T**ogether

A Super-condensed and Reorganized-by-Topic Overview of Beginning Algebra

(Highly abbreviated)

Topics:

Absolute value

The absolute value of a number = take away the negative sign
if there is one. $|-5| = 5$; $|0| = 0$; $|4| = 4$ (p. 296)

Arithmetic of the Integers

Going from –7 to +8 means $8 - (-7) = 15$ (p. 20)

$4 - (-6)$ becomes $4 + (+6)$ (p. 21)

To subtract a negative is the same as adding the positive.

For multiplication:

Signs alike ⇨ *Answer positive*

Signs different ⇨ *Answer negative* (p. 36)

Adding like terms (p. 60)

3 apples plus 3 apples plus 4 apples plus 6 apples plus 2 apples is 18 apples.

A.R.T.

Exponents

$x^2x^3 = (xx)(xxx) = xxxxx = x^5$ (p. 146)

When the bases are the same (that's the number under the exponent), then you *add* the exponents

$(x^2)^3 = x^6$ An exponent-on-an-exponent multiply (p. 155, 307)

$\frac{x^m}{x^n} = x^{m-n}$ (p. 158)

x^{-3} equals $1/x^3$ (p. 158)

x^0 always equals 1. (0^0 is undefined)

Fractional Equations

$$\frac{1}{12} + \frac{1}{16} + \frac{1}{24} = \frac{1}{x}$$ (p. 193)

becomes $\frac{1 \cdot \mathbf{48x}}{12} + \frac{1 \cdot \mathbf{48x}}{16} + \frac{1 \cdot \mathbf{48x}}{24} = \frac{1 \cdot \mathbf{48x}}{x}$

which simplifies to $4x + 3x + 2x = 48$

Memory aid: Santa Claus delivering packages (p. 198)

Fractions

Simplifying: (p. 202)

$$\frac{x^2 + 5x + 6}{x^2 + 6x + 8} = \frac{(x + 3)(x + 2)}{(x + 4)(x + 2)} = \frac{(x + 3)\cancel{(x + 2)}}{(x + 4)\cancel{(x + 2)}} = \frac{x + 3}{x + 4}$$

Memory aid: factor top; factor bottom; cancel like factors

One tricky simplification:

$$\frac{(x - 3)(x - 4)}{x(4 - x)} = \frac{(x - 3)(x - 4)}{-x(x - 4)} = \frac{x - 3}{-x}$$

Adding, subtracting, multiplying & dividing—see p. 202

Long Division by a binomial—see p. 253

For example: $2x + 5 \overline{)\ 6x^3 + 19x^2 + 22x + 30}$

Functions

A function is any rule which associates
to each element of the first set (called the domain)
exactly one element of the second set (called the codomain).

Each element in the domain has an image in the codomain.
The set of images is called the range.
Examples of functions start on p. 260
The identity function maps each element onto itself. (p. 281)

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